The Management of Intelligence-Assisted Finite Element Analysis technology

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Abstract - Artificial Intelligence (AI) approaches to Finite Element Analysis (FEA), have had tentative degrees of success over the last few years and some authors have argued that effective FEA can help in the manufacture reliability and safety aspects of engineered artefacts. The author of this paper reviews how such AI techniques have been applied and in this light, the author then uses a Fuzzy Cognitive Mapping (FCM), to develop a framework for the management of intelligence-assisted FEA.

I. INTRODUCTION

In the design and manufacture of many engineering artefacts, Computer Aided Design (CAD) packages often employ the Finite Element Method (FEM) as a numerical analysis tool for finding physical quantities such as heat, stress and displacement for a particular design. This is achieved by producing an FE (Finite Element) 'mesh' of elements which tessellate together, in a structured or unstructured manner ([29], [36], [44], [70]) within the boundaries of the engineering component model and this process is called Finite Element Mesh Generation (FEMG).

However, the effective modelling and management of FE models and results is, in its simplest form, an arcane task [12]. Software implementations of the method, can sometimes be needlessly verbose and technical, so much so that even experts find difficulty in the modelling of relatively straightforward problems ([10], [12], [30], [52], [68]). Using previous knowledge and interrelationships between model parameters, to overcome such problems does not mean that such a technique is always adaptable to new design scenarios ([15], [16], [27], [46], [51]).

It is evident from the literature that in order to utilise such modelling and computational analysis techniques for the design and manufacture of engineering components, requires some method of harnessing and managing CAD and FEA expertise. Effective use of these computerised approaches to design and analysis should ensure greater reliability and safety of manufactured products from an increased emphasis on design assistance. What is therefore required, is a means by which assistance can be given to the FEM / FEMG process, as and when required [8]. To achieve this, methods such as Knowledge Based Expert Systems [37], Neural Networks [2], Fuzzy Logic ([45], [84]) and Genetic Algorithms ([31], [35]), have attempted to answer such problems and have been tentatively applied to the FEM and FEMG in the last 10-15 years. This paper reviews and outlines how each AI technology has helped in FEA problems that require FEMG, and by the use of a cognitive mapping, a framework for future AI-based FEA is presented.

II. A CLASSIFICATION OF AI METHODS APPLIED TO FINITE ELEMENT ANALYSIS

A classification and taxonomy of each AI method, as discussed previously, is given in Table I with references to Knowledge Based Expert Systems (KBES), Neural Networks (NN), Fuzzy Logic (FL) and Genetic Algorithms (GA) applications.

Expert Systems are generally used to guide the user through the modelling process, by posing questions related to how the user wants to define and analyse the model problem ([33], [48], [55], [56], [76]). A typical KBES architecture for CAD/FEA includes a knowledge 'blackboard' where current information relating to the modelling of the problem is updated, from various solution sources [72].

Fuzzy logic has been used more as an assistant to other applied AI techniques within FE implementations. This has mostly been in the shape of interpreting and quantifying verbose and imprecise definitions of elicited knowledge and constructs and so is only used as a type of data filtering tool ([79], [80]). If combined with a KBES ([67], [77]) it can provide a versatile means to control both engineering judgement and the modelling of the realworld artefact.

Neural Network and Genetic Algorithm implementations have been to a much more limited extent within FEA and FEMG and have mostly been confined to evolutionary growth of an FE mesh and associated shape optimisation([1], [3], [14], [17], [38], [60], [71]). Each paradigm has its advantages and limitations and so any future FEA package/design tool that wishes to achieve the ideals of a perfect FE implementation [10] should try to take into account the feasibility of incorporating such methods based on their *limitations* rather than their advantages [32].

	AI TECHNOLOGY					
CHARACTERISTIC	KBES	NN	FL	GA		
Mechanism	IFTHENELSE	Activation / Threshold	IFTHENELSE	Genetic Operators		
		function (ramp, sigmoid)		(crossover, mutation, etc)		
Envelope	Domain Knowledge	Input data	Closed interval, i.e. [0,1]	Population size		
Advantages	Uses knowledge representation, to reason solution via inference engine	Human brain analogue, ability to learn and recognise patterns	Models vague / imprecise information ('quantifies the unquantifiable')	Finds global minima, obtains best 'natural' solution		
Limitations	Brittle, specialised domain application, long development time required	Network architecture difficult to design , computationally intensive	Modelling is fuzzy in itself, oversimplifies domain knowledge	Computationally intensive, not applicable to all optimisation problems		
Application to FEA	Increase model knowledge representation (Feature Definition) {[11], [15], [16], [22], [24], [28], [33], [39], [40], [49], [50], [54], [[57], [59], [77], [82]}	Quicker convergence of solutions based on 'learnt' Mesh patterns {[1], [3], [15], [16], [18], [38], [42], [58], [71], [81]}	Modelling of vague / imprecise B.C.'s / Geometry {[69], [74], [78], [79], [80]}	Optimisation of model variables (mesh element size, model geometry) {14], [17], [25], [43], [60], [73]}		

TABLE I TAXONOMY OF AI TECHNIQUES APPLIED TO FEA

III. DEVELOPMENT OF A COGNITIVE MAPPING FOR MESH GENERATION

A cognitive mapping for the FEM is now presented, which entails detailing the nature of how each AI technique has been and could be applied to FEMG in particular. This is done so as to develop a framework for incorporating AI within the FEA process (see Section IIIB). The FCM presented here is merely a conceptual mapping not intending to be based on any mathematical background and a more rigorous appraisal of the mathematical basis of FCMs can be found in Simpson [64].

A. An FCM for FEMG

A Fuzzy Cognitive Map (FCM) is a technique that can be used to associate different events and processes via causal rules, which is similar to other 'mind maps' used in psychology ([7], [45]). Essentially, FCMs are nonhierarchic flow charts, within which a series of concepts are linked to other such concepts via associations which are more quantitative than qualitative (i.e. fuzzy / vague linguistic statements such as 'usually', 'not often', 'sometimes').

An FCM which shows the current application of AI techniques to FEM and FEMG, is produced in Figure 1 to elucidate how each technique fits into the FEA process [63]. In what follows, concepts relating to one another are written with their associated fuzzy quantifiers such :

<concept_1> \rightarrow <concept_2> [quantifier]

where the quantifier can be a causal increase or decrease related further to a quantifier (e.g. "sometimes", "maybe", etc). The arrow (\rightarrow) denotes the direction the statement is read in and the FCM can be read in any direction, from any starting node. For example the statement : "Non-linear nodal density distribution \rightarrow Element clustering + always" might be read as "non-linear nodal density distribution always causes an increase in element clustering". The words

'model' and 'problem' relate to FEA scenarios which an analyst may encounter.

From the FCM in Fig.1, the following statements can be produced in a similar fashion in the light of literature which discusses deficiencies of the FEM (references [8],[12], [52], [68]) which are listed in Table II.

TABLE II

a) Problems outside of the knowledge domain may lead to inconsistent / vague assumptions about system variables (i.e. KBES \rightarrow FL - sometimes).

b) KBES's can allow the user to interrogate and define problems within the confines of the knowledge domain, so that geometric representations can lead to successful FE meshes (i.e. KBES \rightarrow FEMG ALGORITHM + geometry).

c) KBES's could assist in steering the solution path of a GA towards realistic and known optima (results), retaining as much design information as possible within the evolved solutions (i.e. KBES \rightarrow GA + sometimes).

d) FL can help to elucidate unquantifiable engineering judgement parameters that can sometimes lead to increased accuracy of modelling the real-world problem (i.e. $FL \rightarrow KBES + mostly$).

e) The imposition of boundary conditions can be easily facilitated using FL to quantify non-linear and irregular phenomena/features which cannot easily be suggested in numerical terms (i.e. $FL \rightarrow FEMG$ ALGORITHM +). f) NN's can assist in the development of a finite element mesh from examples (training data) using self-organising or self-regulating neural architectures (i.e. NN \rightarrow FEMG ALGORITHM +).

g) Optimisation of model problem in terms of either its geometry, finite element nodal distribution, or feature detail, can be achieved by using GAs whereby a geometric (shape) optimisation provides a basis for an effective mesh generation (i.e. $GA \rightarrow FEMG$ ALGORITHM +).

h) A neural	network's	efficiency	can	be	greatly		
improved by	employing a	a GA searc	h for	an	optimal		
neural architecture (i.e. $GA \rightarrow NN + sometimes$).							

B. AI within the FEA process

Combining the FCM presented in Figure 1 with Table II, we can categorise where each AI technology falls within the traditional FEA process as shown in Figure 2. In this process, the user defines a geometric model (the artefact) which is to be analysed by the FEM by producing a mesh for it and subsequently computing and visualising the results (GRAPHICAL VISUALISATION).



Fig. 1 An FCM for Finite Element Mesh Generation via AI-driven paradigms

As an aid to the FEA process, each AI technique assists the user in managing data input and the modelling and analysis of results tasks. This AI-based FEA process increases the understanding of the fundamentals of a problem to be modelled, by the assisted definition of geometry, boundary conditions, and selection of solution controls.

Within the context of Figure 2, a KBES may help to elucidate the required solution technique of the problem (GEOMETRY MODEL), via a known representation for a particular problem e.g. 2D / 3D, linear, bilinear, plate or shell elements (as in PLASHTRAN [11]). In defining the model problem, user knowledge which can help to clarify details of the geometric model could be augmented by fuzzy definitions of boundary conditions, (PHYSICAL MODEL), as Valliapan & Pham have demonstrated [74]. Subsequent changes in the knowledge domain can then be updated by the knowledge of a physical model so that the KBES becomes more of a FKBES (Fuzzy KBES, as in Soh & Yang [67]).

'Intelligent' meshing strategies could encompass notions of mesh adaption, in line with standard numerical measures for mesh adaptivity ([9], [21], [53], [86]) by using NNs or GAs for these tasks (see Table 1, NN and GA columns). The 'intelligence' of such a procedure would lie with locating and selecting those elements in the mesh which had large errors and also with the adaptive procedure itself.

Generating an optimal mesh for the solution of the numerical quantities discussed in Section I is important, since the performance of the FE computation can rely heavily on the 'neatness' of the underlying equations which describe the quantities being sought and the FE tessellation (especially for Kohonen's SOM tessellation [21], [23]). To this end, a GA search of all possible mesh element tessellation / nodal density distributions that provide a low error measure (as described by Babuska & Miller [9] and Zienkiwicz & Zhu [86]), may provide meshes which could be more adaptable than numerically adaptive ones [71]. This is of course at the expense of computational overheads, which for some problems, can be expensive ([7], [31], [42]).

NN or GA techniques aid in the production of an FE mesh remains to be a moot point and a case for further investigation such as in the modal analysis of optical waveguide devices ([20], [62]). The whole FEA picture can be assisted by AI techniques, in the form of managing user information which translates a real world problem to a real world solution. The underlying notion of the process then becomes are of increasing the knowledge of not only the problem being modelled (by elicitation, KBES, and quantification, FL) but also of the generation of approximate domain solutions (by optimisation, NN and GA).



Fig.2 AI within the FEA process (after Finn [22] ; Sharif [61]; Shephard [63])

IV. CONCLUSIONS

A FCM (Fuzzy Cognitive Map) was presented in this paper which helped in showing where each reviewed AI paradigm can have its successful application to Finite Element Analysis, via managing knowledge of the problem being modelled. It was argued that the process of modelling real-world CAD/FEA problems consists of moving from a state of not-knowing enough about how best to model the problem, to knowing how to model it in any future scenarios (as in Figure 2), using elicitation (KBES), quantification (FL) and optimisation (NN or GA) to manage the modelling of an engineering artefact. It is therefore hoped that the emerging AI paradigms looked at in this paper can be organised in such a fashion as to modelling and analysis procedures better via some form of interactive assistance (such as in the development of Problem Solving Environments, [6], [26]).

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